COMMUNICATIONS TO THE EDITOR

Flow Measurements with Ball Meters

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The communication from Shulman and van Wormer (*1*) on this subject appeared while a similar investigation was in progress in this department. Although the correlation equation (4) proposed by these authors is sound, the arguments leading to it are oversimplified. In its equilibrium position the ball comes to rest in contact with the outermost part of the wall of the tube; generally there will be a tangential frictional force acting on the ball at the point of contact.

When the ball is at equilibrium, the moments about its point of contact with the tube wall balance. The tangential component of gravitational force on the ball is ($W \sin \theta$) and acts through the center of the ball. The hydrodynamic force on the ball is f; its resultant will almost certainly be off-center, and so instead of acting through the center of the ball it acts through a point distant X from the wall (Figure 1). Assuming that the radius of curvature of the tube is irrelevant (this is justified by experiment), one finds that the quantities ($f/A_b\rho_f u^2$) and (X/D_b) are functions of (D_b/D_t) and the Reynolds number only When one equates moments,

$$\frac{wD_b}{2}\cdot\sin\theta = fX = A_b D_b \rho_f u^2 \cdot F(Re,\ D_b/D_t) \tag{1}$$

This can be reduced to a form identical with Equation (4) of Shulman and van Wormer, but their empirically determined coefficient C has a significance different from that which they assign to it, unless f acts through the center of the ball.

If f is off-center ($X \neq D_b/2$), the ball will spin at high values of θ because the frictional force required to stop it from spinning exceeds the limit imposed by the coefficient of friction, μ. This will occur when

$$\tan\theta \geq \left|\frac{\mu}{1 - D_b/2X}\right|, \qquad \left(\theta < \frac{\pi}{2}\right) \tag{2}$$

Thereafter a different analysis must be applied. There is thus a *limit to the value* of θ to which the correlation equation (4) can be applied, and the limit depends on the coefficient of friction between the ball and the tube, which may in part be determined by the physical properties of the fluid.

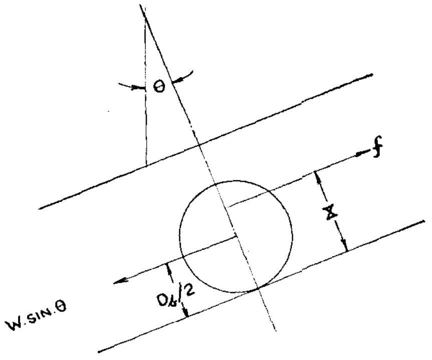

Fig. 1.

The direction of spin will be clockwise (Figure 1) if $X > D_b/2$ and counterclockwise if $X < D_b/2$. If the force f acts through the center of the ball ($X = D_b/2$), the ball will not spin at any value of θ. Observation shows that the ball actually spins clockwise.

LITERATURE CITED

1. Shulman, H. L., and K. A. van Wormer, *A.I.Ch.E. Journal*, 4, 380 (1958).

Reply

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Professor Danckwerts's rigorous treatment of the ball flow meter leads to an equation which is identical with our Equation (4), but it has the advantage of predicting the onset of spinning at high values of θ. We observed this phenomenon and found that at still higher values of θ the ball not only spins but tends to move to and away from the outer wall. When this occurred, the the meter reading θ did not remain constant for a constant flow rate. For this reason, we pointed out, readings were not taken above 60°, and we recommend that the meters be designed conservatively for a midscale reading of 25° to 30°.

The correlation presented applies only when the ball is at rest, that is when it is not spinning. Unfortunately Professor Danckwerts's Equation (2) cannot be used to predict the onset of spinning unless the coefficient of friction is known and a method developed to predict X. Until this information is available, the conservative design method is recommended, and the calculated calibration curve should not be used at values of θ above the observed onset of spinning.

ERRATUM

The caption for Figure 7 in "Heat Transfer to a Liquid Fluidized Bed" by Robert Lemlich and Isidoro Caldas, Jr., appearing on page 377 of the September, 1958, issue, should read: *Effect of flow rate and particle diameter on heat transfer, with methanol as the heating medium.* (*Symbols are identified in Figure 5.*)

The Influence of Heat Transfer on Mass Transfer at Low Pressures

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It is unfortunate that in a number of studies on the evaporation of solids and liquids at low pressures the influence of heat transfer has not been properly taken into account. The result is that the true temperature of the evaporating surface was not used in the analysis of the data and so the reported evaporation coefficients are in error.

A case in point is the very low value for the evaporation coefficient of naphthalene recently reported by Sherwood and Cooke (*5*). This value appears to be low by at least an order of magnitude, owing to a temperature depression of the surface of the evaporating naphthalene sphere which was not considered. The authors indicate that, since the evaporation rate was very small, the evaporative cooling of the surface was negligible. At first glance this seems to be quite reasonable, since only 0.178 g. was evaporated in 1 hr. at the highest rate, and the heat requirement for sublimation amounts to a mere 24 cal. Indeed the very low heat duty of 24 cal./hr. would suggest no problem. However at a pressure of 0.1 μ Hg substantially all heat reaching the sphere must be supplied by radiation from the enclosure walls, and a rather sizable temperature difference is required. With the enclosure maintained at 0°C. the equivalent radiant heat transfer coefficient is about 0.36 cal./(hr.)(sq. cm./°C.). (This is the maximum coefficient when all emissivities are assumed to be 1.0.) Hence the sphere, 1.0 cm. in diameter, evaporating at a pressure of 0.1 μ, would tend to attain a steady-state surface temperature of about −21°C. It is to be noted that the vapor pressure of naphthlene at −21°C. is about 0.38 μ Hg compared with 5.74 μ Hg at 0°C.

With the true vapor pressure of the surface introduced into Equation (1) of their paper and the evaporation coefficient assumed to be unity, the maximum evaporation rate at a pressure of 0.1 μ Hg would be about 0.177 g./hr. This agrees almost exactly with the observed result. Thus it appears that the evaporation coefficient of naphthalene is in reality close to unity, and the low apparent value reported (about 0.056) merely reflected the temperature depression of the naphthalene surface. The same argument also suggests that the true evaporation coefficient of diethyl adipate is close to unity, rather than 0.177 as reported.

While the largest temperature depression of the surface was encountered in the experiments at the lowest pressure, 0.1 μ, where heat transfer to the sphere was most unfavorable, some temperature depression also was involved at the higher pressures studied. This temperature depression tended to decrease with increasing pressure for two reasons: (1) increased diffusional resistance in the gas reduced the evaporation rate and hence the required heat duty, and (2) heat transfer to the sphere was improved at the higher pressures because of increased heat conduction through the gas.

As the evaporation of spheres (or drops) has been studied in a number of cases, a general equation has been developed for predicting the temperature depression of the surface accompanying evaporation into a quiescent gas at low pressures. The equation is shown to reduce to simplified forms for the limiting cases of very low pressures and relatively high pressures. The term *relatively* high pressures is used, as the pressure must be low enough to assure the absence of convection. As a rough guide, the upper pressure limit would be of the order of 1 mm. Hg.

EVAPORATION OF SPHERES AT LOW PRESSURES

The evaporation rate for a sphere in a gas at low pressures has been considered in detail by Bradley, Evans, and Whytlaw-Gray (*1*). Considering the enclosure to be very large and the concentration of the evaporating substance to be zero at the walls of the enclosure, one can develop the following equation for the rate of evaporation:

$$-\frac{dm}{dt} = \frac{4\pi a D_v C_s}{\dfrac{D_v}{\alpha_M \Upsilon a} + \dfrac{a}{a + \Delta}} = \frac{4\pi a^2 C_s}{\dfrac{1}{\alpha_M \Upsilon} + \dfrac{a^2}{(a + \Delta) D_v}} \qquad (1)$$

This equation was developed on the basis of an evaporation model proposed by Fuchs (*2*). In his treatment evaporation from the sphere occurs through a surrounding vacuous shell of thickness Δ at the theoretical vacuum rate. In the steady state, molecules arriving at the shell surface at a distance $a + \Delta$ from the sphere center leave by diffusion through the gas.

Under high vacuum conditions, where $\Delta \gg a$, and $a/(a + \Delta) \rightarrow 0$, this equation reduces to the familiar form for the evaporation rate in an absolute vacuum:

$$-\frac{dm}{dt} = 4\pi a^2 \alpha_M \Upsilon C_s \qquad (2)$$

HEAT TRANSFER TO A SPHERE AT LOW PRESSURES

When one follows the evaporation model of Fuchs, heat conduction to the sphere may be assumed to take place in the following manner. Conduction occurs through the gas up to the outer surface of the vacuous shell and then by free molecule conduction across the vacuous shell to the sphere surface. A similar argument was used previously by Kyte, Madden, and Piret (*4*).

The heat flow to the sphere by conduction through the gas will then be given by an equation analogous to Equation (1),

$$\frac{dQ_c}{dt} = \frac{4\pi a^2 (T_\infty - T_s)}{\dfrac{1}{\alpha_H \Lambda_0 P \sqrt{\dfrac{T_s}{273}}} + \dfrac{a^2}{(a + \Delta)k}} \qquad (3)$$

At low pressures, where $\Delta \gg a$, Equation (3) reduces to

$$\frac{dQ_c}{dt} = 4\pi a^2 \alpha_H \Lambda_0 P \cdot \sqrt{\frac{273}{T_s}} (T_\infty - T_s). \qquad (4)$$

This will be recognized as the equation for heat transport by free molecule conduction. At the other extreme, where the gas pressure is relatively high, $\Delta \ll a$, Equation (3) reduces to

$$\frac{dQ_c}{dt} = 4\pi a k (T_\infty - T_s) \qquad (5)$$

the usual equation for heat conduction to a sphere through a surrounding infinite medium.

The total heat transfer to a sphere will include simultaneous radiant heat transfer. Hence the total heat transfer may be represented by

$$\frac{dQ}{dt} = \left[h_r + \frac{1}{\dfrac{1}{\alpha_H \Lambda_0 P \sqrt{\dfrac{T_s}{273}}} + \dfrac{a^2}{(a + \Delta)k}} \right] \cdot 4\pi a^2 (T_\infty - T_s) \qquad (6)$$

HEAT BALANCE

In the steady state the heat transfer to the sphere will satisfy the latent heat requirement for evaporation (or sublimation). Hence

$$-\frac{dm}{dt} = \frac{1}{L_v} \frac{dQ}{dt} \qquad (7)$$

thus

$$\frac{4\pi a^2 C_s}{\dfrac{1}{\alpha_M \Upsilon} + \dfrac{a^2}{(a + \Delta) D_v}}$$